PERIODIC REVIEW INVENTORY MANAGEMENT WITH BUDGET CONSTRAINTS: DISCRETE-EVENT SIMULATION AND SENSITIVITY ANALYSIS

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KEYWORDS

Inventory management, periodic review, budget constraint, discrete event simulation, procurement lead time

ABSTRACT

In this study, a periodically reviewed inventory system with an order-up-to-level policy and budget constraints is considered. A discrete-event simulation model is developed to identify the optimal operating conditions, in terms of reorder period (DT) and order up to level (OUTL). Two different scenarios, characterized by different unit costs, are considered. A sensitivity analysis is finally conducted to evaluate the impact of procurement lead time and budget constraints on optimal conditions. A linear correlation between optimal DT and optimal OUTL is found and the equation of the line is derived as a function of the average daily demand and procurement lead time.

INTRODUCTION

Good inventory management is essential, on the one hand, to reduce the management cost and, on the other hand, to ensure high customer satisfaction. The most critical and challenging aspect of inventory management, indeed, is balancing supply and demand so that products are not overstocked or in shortage. Keeping inventory at a low level, indeed, can reduce the costs of keeping stock on hand, but at the same time increases the risk of shortages. On the contrary, maintaining inventory at a high level reduces the risk of shortages, but increases the cost of storage. “How much to order?" and "When to order?" are the two biggest problems that managers must solve when they manage an inventory.

Several approaches can be found in the literature aimed at optimizing inventory management. Some authors focus on economic optimization, by both minimizing the total management cost (Jeenanunta et al., 2021; Kouki et al., 2014; Qiu et al., 2022) or maximizing the profit (Çomez & Kiessling, 2012; Zhang, 2012; Zhang et al., 2008). Generally, the cost items considered are the stock-keeping cost, the order-issuing cost, and the stock-out cost. The latter, when backlogs are admitted, coincides with a shortage cost; in other cases, out-of-stock situations are considered to cause lost sales.

Other authors focus on the customer satisfaction point of view by introducing service level constraints to maximize the level of demand satisfaction (Chen & Li, 2015; Minner & Transchel, 2010; Wang & Chen, 2022).

In real scenarios, making these decisions is a great challenge especially because constraints caused by limited resources have to be faced. One of the most important constraints imposes budget restrictions on the amount that can be spent on stocks. This constraint is studied in different fields using different methods. Bera et al. (2009) have presented a continuous-review inventory model with a budget constraint in which the purchase cost of the system is paid when an order comes.

Also, available storage space is often a constraint to be met. Hariga (2010) has presented a continuous review inventory system with constraints of stochastic space for a single item and random demand in which the quantity of the order and reorder point are decision variables. Finally, also lead time can be considered as a time constraint that affects system performance and should be taken into account to optimize management policy (Ben-Daya & Raouf, 1994). Depending on the case, it is treated as a predetermined parameter constant or stochastic. To the best of the authors' knowledge in the literature there are no studies that address the optimization of periodically reviewed inventory management systems with budget constraints using a simulation approach. Furthermore, no study evaluates the impact that budget constraint amount, procurement lead time as well as unit costs have on the optimal inventory management policy.

In this study, a discrete-event simulation model of an order-up-to-level periodic review policy with a budget constraint was developed. The simulation model was then used to identify the optimal combination of the operating leverages for minimizing the total management cost. Finally, a sensitivity analysis was conducted to assess the impact that both budget constraints and lead time have on management policy.

The article is organized as follows: the model overview as well as nomenclature and assumptions are reported in section 2; numerical simulation results are presented and discussed in section 3. Finally, in section 4, conclusions are reported and future research activities are outlined.

MODEL OVERVIEW AND ASSUMPTIONS

Table 1 – Nomenclature

Symbol	Description	Unit
i	i-th day (i = 1,…,n)	-
ΔT	Reorder period	days
$OUTL$	Order-up-to-level	kg
LT	Procurement lead time	days
OH_i	On-hand inventory on the day i	units
O_i	Quantity purchased on the day i	kg
$\overline{D}$	Average daily demand	kg/day
σ	Standard deviation of the daily demand	kg/day
OOS_i	Out-of-stock on the day i	kg/day
$C_{oi,i}$	Order-issuing cost on the day i	€/order
c_{so}	Unitary stock-out cost	€/day
c_{inv}	Unitary inventory holding cost	€/kg
c_p	Unitary purchase cost	€/units

The model assumes a normally distributed demand with a known mean and standard deviation. Unitary costs are assumed to be constant and deterministic. Two different scenarios characterized by different unitary costs were considered, as reported in Table 2. The impact of procurement LT was assessed by considering three LT values for each scenario (1, 2, and 3 days). In the case of stock-out situations, shortages are allowed and fully back-ordered.

Table 2 - Operating conditions of the two simulated scenarios

	Scenario 1	**Scenario 2**
$\overline{D}$	2000	2000
σ	100	100
c_p	2	2
c_{oi}	1500	1500
c_{inv}	0.025	0.050
c_{so}	0.25	0.50

According to the order-up-to-level periodic review policy, the stock on hand is periodically reviewed and, the orders are issued at constant time intervals (ΔT) to restore the target level of stock, i.e., order-up-to-level (OUTL). In this paper, budget constraints are included in the problem formulation, resulting in an upper limit in the quantity that could be ordered each time. Unlike traditional periodic review policy, therefore, it might happen that the maximum orderable quantity is not sufficient to restore the desired stock level. This limitation increases the risk of stock-out conditions, generating the need of reviewing the inventory management policy to adjust the levels of operating leverages ΔT and OUTL based on the system constraints. Operating leverages must therefore necessarily be determined by keeping in mind the budget constraint to maximize system performance.

The simulation model was then used to simulate a sufficient number of periods (days), 50’000 in this study, to consider different combinations of the operating leverages and identify the one that minimizes the total management cost while respecting the budget limit. For each scenario, the reordering period was varied between LT and 10 and the target OUTL between 1000 and 30000, assuming the latter to be a multiple of 500. In addition, to assess how much the budget constraint impacts the system's optimal operating conditions, for each configuration, the budget constraint was varied between 10’000 € and 1’000’000 €. For each period the following activities are performed:

(i) If an order is scheduled for the day i a number of items such as to restore OUTL or, if the budget limit prevents us from ordering this quantity, the maximum quantity of items that can be purchased with the available budget, is ordered:

$$O_i = \min\left(OUTL - OH_i; {}^{Budget}/_{c_p}\right) \quad (1)$$

(ii) If an order was issued LT days before, the stock is updated according to the quantity ordered:

$$OH_i = OH_{i-1} + O_{i-LT} \quad (2)$$

(iii) If OH_i meets demand at day i (d_i) is fulfilled and OH_i is consequently updated:

$$OH_i = OH_i - d_i \quad (3)$$

Otherwise, If OH_i doesn't meet d_i, out of stock on day i is computed:

$$OOS_i = d_i - OH_i \quad (4)$$

(iv) The inventory holding cost is determined based on the quantity in stock:

$$C_{inv,i} = c_{inv} \cdot OH_i \quad (5)$$

(v) Stock-out cost is a cost that occurs when the inventory level is not sufficient to satisfy customers' demand:

$$C_{so,i} = c_{so} \cdot OOS_i \quad (6)$$

(vi) The purchase cost is strictly dependent on the quantity purchased:

$$C_{p,i} = c_p \cdot O_i \quad (7)$$

(vii) The order issuing cost $C_{oi,i}$ is fixed and independent of the quantity ordered.

(viii) The total cost of the system is given by the sum of the described cost items:

$$C_{tot,i} = C_{inv,i} + C_{so,i} + C_{p,i} + C_{oi,i} \quad (8)$$

The overall structure of the model is presented in Figure 1.

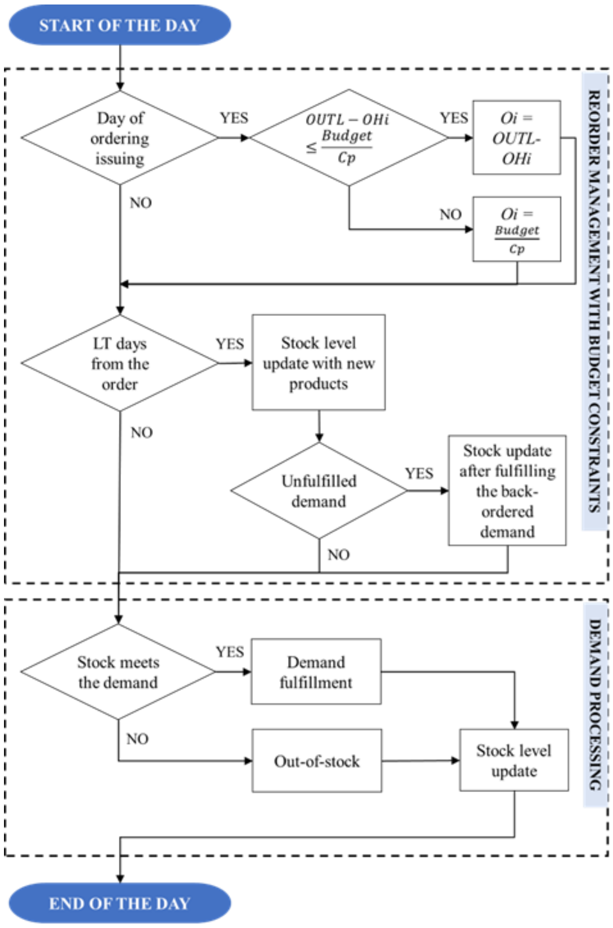

Figure 1 – Flowchart of the proposed approach

RESULTS AND DISCUSSION

The optimal operating conditions, in terms of DT and OUTL, with the associated total cost as the budget constraint changes, for scenario 1 and scenario 2, considering LT=1, are shown in Tables 3 and 4, respectively.

For both scenarios, the optimal operating condition remains unaffected by the budget constraint until the budget value is sufficient to guarantee the feasibility of the unconstrained optimum point, (i.e., 8 days as DT and 16'000 units as OUTL for scenario 1 and 6 days as DT and 12'000 units as OUTL for scenario 2). For lower budget values, the optimal DT and OUTL decrease, resulting in more frequent orders of smaller quantities, and the related total cost increases.

By observing the results obtained for each scenario, the optimal points were found to present a clear linear trend: in particular, it can be noted that as the budget constraint increases, the operating leverages increase

Table 3 - Optimal operating leverages as the budget constraint varies for Scenario 1 and LT1

Budget [€]	**DT** [days]	**OUTL** [units]	**C_{tot}** [€]
1'000'000.00	8	16000	4365.24
100'000.00	8	16000	4365.24
80'000.00	8	16000	4365.24
70'000.00	8	16000	4365.24
65'000.00	8	16000	4365.24
60'000.00	8	16000	4365.24
55'000.00	8	16000	4365.24
50'000.00	8	16000	4365.24
45'000.00	8	16000	4365.24
40'000.00	8	16000	4365.24
35'000.00	8	16000	4365.24
30'000.00	7	14000	4366.79
25'000.00	6	12000	4378.19
20'000.00	4	8000	4454.30
15'000.00	3	6000	4555.01
10'000.00	2	4000	4781.59

and the value of the minimum cost decreases (Figure 2). Moreover, when LT increases, since when the order is issued, the quantity in stock is higher as it must meet the demand of *(LT-1)* days (same-day demand has already been met), the optimal value of OUTL is higher.

On the other hand, the optimal DT does not result to be affected by lead time. The theoretical on-hand inventory (dashed line in Figure 3), which is updated at the moment the order is issued, reflects the optimal OUTL value, as it accounts for the quantity ordered and for the demand expected during the procurement LT period. On the other hand, the real stock-on-hand pattern (solid line in Figure 3) is only shifted forward by LT days. The average quantity ordered, indeed, appears to remain almost the same independently from LT.

Looking at the results of scenario 2, it appears that both the optimal DT and the optimal OUTL decrease.

Table 4 - Optimal operating leverages as the budget constraint varies for Scenario 2 and LT1

Budget [€]	**DT** [days]	**OUTL** [units]	**C_{tot}** [€]
1'000'000.00	6	12000	4507.65
100'000.00	6	12000	4507.65
80'000.00	6	12000	4507.65
70'000.00	6	12000	4507.65
65'000.00	6	12000	4507.65
60'000.00	6	12000	4507.65
55'000.00	6	12000	4507.65
50'000.00	6	12000	4507.65
45'000.00	6	12000	4507.65
40'000.00	6	12000	4507.65
35'000.00	6	12000	4507.65
30'000.00	6	12000	4507.65
25'000.00	6	12000	4507.65
20'000.00	4	8000	4534.72
15'000.00	3	6000	4611.32
10'000.00	2	4000	4814.29

Indeed, since both c_{inv} and c_{so} are higher, it is more convenient to order, more frequently, a lower average amount of items. This reduces both the stock-out probability and the average stock level. The correlation between the optimal DT and the optimal OUTL, as the budget constraint changes, is confirmed to be linear.

This correlation was fitted with a linear model and the equation of the line was then derived (equation 9).

$$OUTL_{opt} = \overline{D} \cdot DT + \overline{D} \cdot (LT - 1) \qquad (5)$$

It can be observed that the procurement LT affects the intercept, while it does not affect the slope of the line, which is only affected by the average daily demand (Figure 4 and Figure 5).

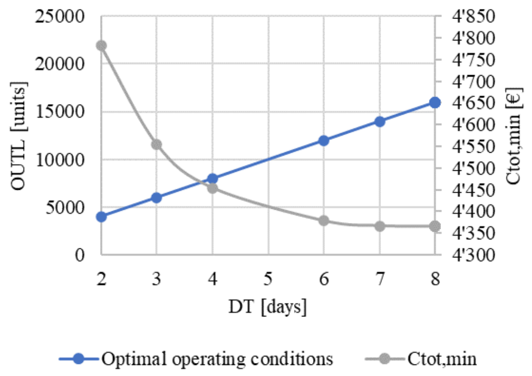

Figure 2 - Trend in optimal operating conditions and total cost values for different budget constraints for Scenario 1 and LT1

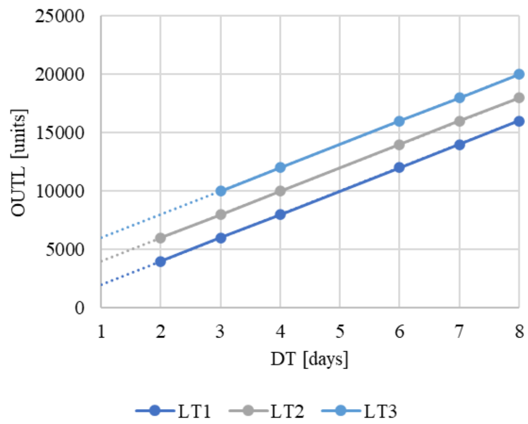

Figure 3 – Scenario 1; trend of the optimal combinations of the operating leverages DT and OUTL

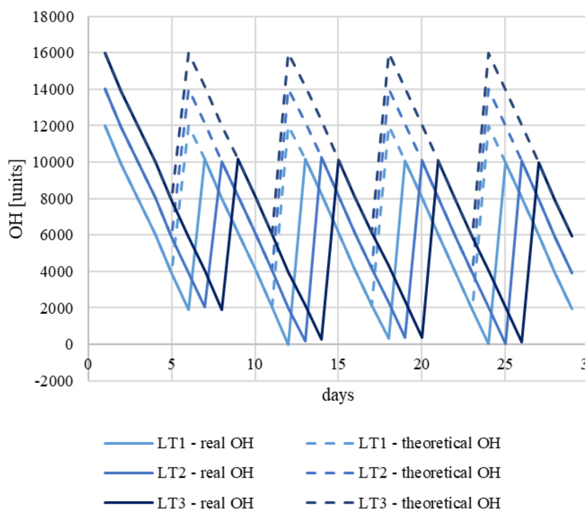

Figure 2 – OH levels in the case of Scenario 2 and three different values of procurement LT values (1, 2, and 3)

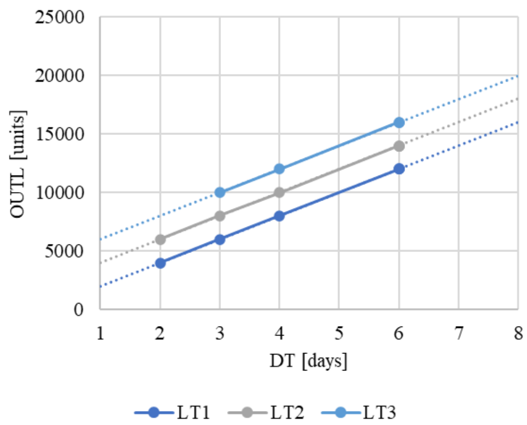

Figure 4 – Scenario 2; trend of the optimal combinations of the operating leverages DT and OUTL

CONCLUSIONS

A discrete event simulation model of an order-up-to-level periodic review policy with a budget constraint was developed.

The simulation model was used to identify the optimal operating condition, in terms of the combination of DT and OUTL, to minimize the total management cost system while meeting the budget constraint.

A sensitivity analysis was then performed to assess how the optimal condition varies as LT, budget constraint, and unit costs (c_{inv} and c_{so}) vary.

Results highlighted that, as the lead time changes, with all other parameters remaining constant, the value of optimal OUTL changes proportionally to the lead time value, while the value of optimal DT does not change.

A linear trend describing the correlation between DT and OUTL was derived, which appears to be strongly influenced by average demand.

In future studies, it will be interesting to evaluate the impact of different costs and system parameters on the correlation identified and described in this study. A simulative campaign based on DOE will also be interesting to assess the significance of each parameter on the output response of the model (i.e. total management cost).

REFERENCES

Ben-Daya, M., & Raouf, A. (1994). Inventory Models Involving Lead Time as a Decision Variable. *The Journal of the Operational Research Society*, *45*(5), 579. https://doi.org/10.2307/2584393

Bera, U. K., Rong, M., Mahapatra, N. K., & Maiti, M. (2009). A multi-item mixture inventory model involving random lead time and demand with budget constraint and surprise function. *Applied Mathematical Modelling*, *33*(12), 4337–4344. https://doi.org/10.1016/j.apm.2009.03.025

Chen, H., & Li, P. (2015). Optimization of (R, Q) policies for serial inventory systems using the guaranteed service approach. *Computers & Industrial Engineering*, *80*, 261–273. https://doi.org/10.1016/j.cie.2014.12.003

Çomez, N., & Kiessling, T. (2012). Joint inventory and constant price decisions for a continuous review system. *International Journal of Physical Distribution & Logistics Management*, *42*(2), 174–202. https://doi.org/10.1108/09600031211219672

Hariga, M. A. (2010). A single-item continuous review inventory problem with space restriction. *International Journal of Production Economics*, *128*(1), 153–158. https://doi.org/10.1016/j.ijpe.2010.06.008

Jeenanunta, C., Kongtarat, V., & Buddhakulsomsiri, J. (2021). A simulation-optimisation approach to determine optimal order-up-to level for inventory system with long lead time. *International Journal of Logistics Systems and Management*, *38*(2), 253. https://doi.org/10.1504/IJLSM.2021.113250

Kouki, C., Jemai, Z., Sahin, E., & Dallery, Y. (2014). Analysis of a periodic review inventory control system with perishables having random lifetime. *International Journal of Production Research*, *52*(1), 283–298. https://doi.org/10.1080/00207543.2013.839895

Minner, S., & Transchel, S. (2010). Periodic review inventory-control for perishable products under service-level constraints. *OR Spectrum*, *32*(4), 979–996. https://doi.org/10.1007/s00291-010-0196-1

Qiu, R., Sun, Y., & Sun, M. (2022). A robust optimization approach for multi-product inventory management in a dual-channel warehouse under demand uncertainties. *Omega*, *109*, 102591. https://doi.org/10.1016/j.omega.2021.102591

Wang, L., & Chen, H. (2022). Optimization of a stochastic joint replenishment inventory system with service level constraints. *Computers & Operations Research*, *148*, 106001. https://doi.org/10.1016/j.cor.2022.106001

Zhang, J.-L. (2012). Integrated Decision On Pricing, Promotion And Inventory Management. *Asia-Pacific Journal of Operational Research*, *29*(06), 1250038. https://doi.org/10.1142/S0217595912500388

Zhang, J.-L., Chen, J., & Lee, C.-Y. (2008). Joint optimization on pricing, promotion and inventory control with stochastic demand. *International Journal of Production Economics*, *116*(2), 190–198. https://doi.org/10.1016/j.ijpe.2008.09.008

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