# **ANALYSIS OF THE HOUSING MARKET DYNAMICS USING NARX NEURAL NETWORK**

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## **KEYWORDS**

NARX model, Nonlinear Autoregressive Exogenous Model, time series prediction, machine learning, real estate forecast.

# **ABSTRACT**

This study employs a Nonlinear Autoregressive with eXogenous inputs (NARX) neural network to model the dynamics of the housing construction market in Poland, with a distinction made between segments of developers and individual investors. The dataset under analysis contains the 19-year data corresponding to the numbers of housing units approved for construction, under construction, and completed. The NARX model was calibrated thoroughly to suit unique characteristics of the data, with an emphasis put on the hidden layer size and delay parameters, to capture the estate market's nonlinear trends. Results show a very high efficiency of NARX models and highlight distinct patterns and dynamics in the housing completion, construction starts, and permit issuance between the two market segments. These variations are vital for understanding the distinct forces and trends shaping the developers' and individual investors' markets in the Polish housing sector. Findings of the analysis provide valuable insight into the nuanced functioning of these market segments.

## **INTRODUCTION**

The real estate market plays a crucial role in the economy, representing a key sector that impacts numerous other industries (Breuer and Steininger, 2020; Gomez-Gonzalez et al., 2024). First of all, the real estate market, particularly the construction and real estate trade, generates significant employment opportunities, engaging both construction professionals and service sector representatives (Xiong, 2023). Second, the real estate market influences credit activity, serving as an important income source for financial institutions while affecting the financial balance of households at the same time (Lang et al., 2022). Last, property values directly impact consumer wealth, shaping their capital and affecting their ability to invest and utilize various

financial instruments (de Bondt et al., 2020; Hong, 2014). The real estate market in Poland is relatively stable although it has slowed down in recent years, mainly due to a very high inflation, high-interest rates, and insecure geopolitical situation (Statista, 2024).

The pivotal role of the real estate market underscores the importance of a precise identification of changes occurring within it. Thus, accurate determination of its parameters is crucial for effective management in this market. In practice, to identify parameters of this market, time series data or panel data is utilized and various analytical methods are applied (Shao et al., 2020). Commonly used methods include mathematical (especially statistical) and artificial intelligence-based techniques (Grybauskas et al., 2021; Kakulu, 2014). In recent years the use of artificial neural networks in real estate market analysis has been particularly popular (Gabrielli et al., 2021; Frącz et al., 2023), including the application of NARNET models (Wotzka et al., 2023). In this context, this paper continues the previous research, demonstrating the efficient use of the NARX Neural Network tool (MathWorks, 2024). A research hypothesis underlying our study assumes that NARX NN can be a reliable tool to model time series representing three main components of the housing construction market: the issuance of house building permits, the house construction, and completion of the house construction, with a distinction made between developers and individual investors.

## **DATA AND RESEARCH METHODS USED**

## **Time Series Data**

The data subjected to analysis was the free data made available by the Central Statistical Office in Poland (GUS, 2023). The data covers a period of more than 19 years (228 months), ranging from January 2005 to December 2023. Six datasets were used in our study, including the numbers of house building permits issued to individual investors and to developers, the numbers of houses under construction by individuals and developers, and the numbers of houses completed and put into use by these two groups.

Each dataset is represented as a univariate time series, where an element of a series represents the number of houses under consideration in a given time slot (month of a year). Thus, six time series were prepared:

- *permits-ind* house building permits issued for individual investors,
- *permits-dev* house building permits issued for developers,
- *initiated-ind* houses under construction (constructions started) by individual investors,
- *initiated-dev* houses under construction by developers,
- *completed-ind* houses that have been completed by individual investors,
- *completed-dev* houses finished by developers.

There were two outliers in *completed-ind* series and one outlier in *completed-dev* series. These outliers were replaced by respective means. Basic information on the time series used are given in Table 1.

Dataset	Avg.	Min.	Max.	Std. dev.	Var. coeff.	Corr.
<i>permits-ind</i>	7817.2	3201	12010	1855.4	23.7	0.1
<i>permits-dev</i>	9477.9	1608	26076	4647.3	49.0	0.7
initiated-ind	6993.7	1236	11958	2572.2	36.8	0.1
initiated-dev	7256.4	1149	17821	3494.9	48.2	0.7
completed-ind	5713.2	2329	10518	1169.7	20.5	0.5
completed-dev	7344.0	1665	16793	3338.6	45.5	0.8

Table 1: Basic Statistics and Correlations of Time Series

## **Methods**

## *Z-Score Standardization*

To enable the comparison between the series, deal with outliers, and ensure precise and consistent interpretation of results, all the data was standardized using the Z-Score method. This technique uses the average and the standard deviation of each variable to adjust the corresponding data points to have a mean of zero and a standard deviation of one. The transformed data points reflect their deviation from the mean, with positive zscores indicating values above the mean and negative scores indicating values below it.

### *NARX NN*

To model the dynamics of the housing construction market we are utilizing a NARX (Nonlinear Autoregressive with eXogenous inputs) network with various configurations. We focus on assessing the effectiveness of different combinations of hidden layer sizes, training algorithms, and delays in the context of the normality of prediction residual distributions.

Traditional autoregressive models, such as AR, ARMA, and ARIMA, rely on linear relationships between successive data points. These are statistical models, in which future values of a time series are predicted based on a combination of its past values. In contrast, autoregressive neural networks are able to model complex, nonlinear relationships in time series data. An autoregressive NN consists of layers of neurons that receive inputs from previous time points. The model allows one to adjust the *delay* window length, affecting how many previous time steps are taken into account in making predictions. The network is trained to predict current time series values based on the historical data. During the learning process, an error between predictions and actual values (MSE – *Mean Squared Error*) is minimized by modifying the network's weights. NARX NN is an advanced method for modeling and forecasting which utilizes nonlinear autoregressive models with external inputs. It is widely used in various scientific fields due to its ability to model complex nonlinear relationships. A mathematical equation of the NARX model is the following:

$$
\mathbf{Y}(t) = f(\mathbf{Y}(t-1), \mathbf{Y}(t-2), ..., \mathbf{Y}(t-d_{y}),
$$
  
 
$$
\mathbf{X}(t-1), \mathbf{X}(t-2), ..., \mathbf{X}(t-d_{x})) + e(t),
$$

where  $Y(t)$  – the observed value of the dependent variable at time *t* (model output),  $X(t)$  – a vector of values of the independent variable at time *t* (model input), *d<sup>y</sup>* and *d<sup>x</sup>* – delays for the dependent and independent variables, respectively,  $e(t)$  – a model error or noise, representing unaccounted factors,  $f(\cdot)$  – a nonlinear function which can be realized using various techniques, such as neural networks.

The number of neurons in the input layer equals to the sum of both delays:  $d_y + d_x$ . The output of the network constitutes of single neuron, which can be mathematically described as:

$$
y(t) = \sum_{i=1}^{HL} v_i \cdot h_i + b_{output}
$$
  

$$
h_i = tansi g\left(\sum_{j=1}^{d_y + d_x} w_{ji} \cdot input_j + b_i\right)
$$

where  $d_y + d_x$  – the number of neurons in the input layer,  $w_{ij}$  – the weight for the *i*-th neuron of the hidden layer from the *j*-th neuron from the input layer *input<sub>i</sub>*,  $b_i$  – the *i*-th bias,  $h_i$  – an activation function of the *i*-th neuron in the hidden layer (we apply *tansig*, i.e., a hyperbolic tangent sigmoid transfer function), *HL* – the number of neurons in the hidden layer,  $v_i$  – the weight of the output layer from the *i*-th neuron of the hidden layer, *boutput* – an output bias. Furthermore, we apply a linear transfer function at the output layer and assume  $d_x = d_y$ .

A schematic view of an example NARX network is given in Fig. 1. This network comprises a single hidden layer with 30 neurons. Delays  $d_x$  and  $d_y$  are marked within the hidden layer as "1:150".

Our idea for improving effectiveness of model predictions involved creating matrix **X** and vector **Y** as described in the following code listing:

ModelOrder = 18; X = []; Y = [];

- for i = ModelOrder + 1:length(timeseries)
- $X = [X;$  timeseries(i ModelOrder: i 1)'];

Y = [Y; timeseries(i)];end

An order of the autoregressive model (i.e. the size of matrices **X**) equal to 18 had been selected at the initial stage of the study. We carried out preliminary experiments for model order values ranging from 1 to 18 (based on the size of the input time series, containing 228 elements). Models of the 18th order demonstrated the lowest values of MSE. This means that the **X** matrix contains 18 sequences of data, each shifted back by one time interval (one month), thereby implementing the regression. Moreover, the standard input for the model is the same time series sequence, **Y**.



Figure 1: A Schematic View of an Example NARX Network with *delay* = 150, order = 18, and *HL* = 30.

Other parameters of the NARX model, including delays  $d<sub>y</sub>$  and  $d<sub>x</sub>$ , as well as the form of function *f*, have been selected based on characteristics of the data and the specificity of the problem the model is intended to solve. A proper selection of these parameters is crucial for the high effectiveness and accuracy of the model.

In experiments three hidden layer sizes were investigated: *HL* = 10, 20, 30. Values of *delay* (i.e., time steps), were the following: 1, 5, 10, 25, 50, 75, 100, 125, 150. Eight learning functions were applied:

- CGD Conjugate Gradient with Powell/Beale Restarts,
- CGF Fletcher-Reeves Conjugate Gradient,
- GDA Gradient Descent with Adaptive Learning Rate,
- GDM Gradient Descent with Momentum,
- GDX Gradient Descent with Momentum and Adaptive Learning Rate,
- OSS One Step Secant,
- RP Resilient Backpropagation,
- SCG Scaled Conjugate Gradient.

The learning process was conducted in 500 iterations, which allowed us to maintain the minimum value of MSE. Extending the learning process beyond 500 iterations did not improve the results significantly.

### *The Jarque-Bera (JB) Test*

After conducting the regression process, a well-known practice to examine the quality of a model is investigation of the normality of residuals. The Jarque-Bera (JB) test is a popular statistical test used to check whether residuals follow a normal distribution. It is based on two measures of the distribution of the variable under study – the skewness and the kurtosis:

$$
JB = n/6 (\gamma^2 + 0.25 (K - 3)^2),
$$

where: *n* is the number of observations in the sample, *γ* is the skewness, *K* is the kurtosis. The JB statistics adheres to a chi-square distribution with two degrees of freedom. When the p-value (probability) associated with this statistics falls below the predetermined level of significance ( $\alpha = 0.01$  in our study), the hypothesis of data normality is rejected (in this case we denote the results as  $H = 1$ ). Otherwise, we lack grounds to reject this hypothesis (and the result is  $H = 0$ ). Outcomes of these analyses are detailed in the subsequent section.

## **ANALYSIS OF MODELING RESULTS**

Performance of the developed NARX NN models was evaluated based on MSE across varying parameters: delay, the hidden layer size, and a learning function. Due to the space limit we do not present graphical results for all the tested cases but only for best models. Example model responses are presented in Fig. 2 (for the *permits-ind* scenario) and Fig. 3 (for the *permits-dev*



Figure 2: Response of NARX with CGF Learning Algorithm, *HL* = 30, *delay* = 100 for *permits-ind* Data



Figure 3: Response of NARX with CGF Learning Algorithm, *HL* = 30, *delay* = 100 for *permits-dev* Data

The upper part of each figure visualizes the original data along with the model response to the training data and the model response to the test data. One can observe that for both scenarios the time series exhibit a clear seasonality, which is highly related to the seasons of a year. Seasonal patterns for developers and individuals are different, however, and a plot representing building permits issued to developers has a more evident increasing trend. In both cases the model efficiency is very high; it is confirmed by low MSE values (visible above the plots), which are below 0.04.

Lower parts of Fig. 2 and Fig. 3 contain histograms of residuals obtained in both the training (bottom-left) and the testing (bottom-right) processes, along with a fitted normal distribution curve. Values of the Jarque-Bera test are given as well, showing whether the hypothesis of normality of the calculated residuals is accepted  $(H = 0)$ or rejected  $(H=1)$ . Such a presentation aims at providing a comprehensive evaluation of the model performance, emphasizing the analysis of residuals as an important component of model validation and underscoring the importance of statistical testing in confirming assumptions underlying the model's application.

Table 2 summarizes the best NARX models obtained for all six datasets (these are models for which the hypothesis of residuals' normality according to the Jarque-Bera (JB) test is not rejected). It can be seen that the efficiency of all the models is very high, resulting in very low MSE. In most cases, the best results were achieved by employing Conjugate Gradient with Powell/Beale Restarts training algorithm (CGD). In contrast, regarding the number of neurons in the hidden layer, one cannot generalize the best value of *HL* parameter. Regarding the delay (the number of previous time steps taken into account in making predictions), it can be observed that at least 100 steps should be taken into consideration when building an efficient model.

To better assess the model performance and fit, subsequent analysis results are depicted in Fig. 4, containing the best model cases for all six time series (results for the best models are marked in Fig. 4a – Fig. 4f with black ovals).

Plots on the left side of the figure provide detailed graphical representations of performance metric MSE achieved by the respective NARX models over 500 training iterations. These charts illustrate in detail how MSE varies depending on the delay for each specified learning algorithm, offering a clear visual depiction of their performance characteristics. It should be emphasized that characteristics and tendencies of individual algorithms were consistent for all test cases (all three hidden layer sizes), also for these not shown in the figure.

Plots on the right side of Fig. 4 visualize the corresponding results of the Jarque-Bera test for various training algorithms and delays, showing whether the hypothesis of data normality is rejected  $(H = 1)$  or not  $(H = 0)$ .



Table 2: MSE and Parameters for NARX Models with Best Performance

It can be observed that Resilient Backpropagation algorithm (RP) practically does not work for the problem under consideration – it is insensitive to delay changes and is not capable of providing low MSE. Gradient Descent-based algorithms (GDM, GDA, and GDX), as well as Fletcher-Reeves Conjugate Gradient (CGF) tend to improve slightly with the delay increase but they are not able to provide acceptable MSE even for very large delays. Efficiency of other training algorithms (CGD, SCG, OSS) varies across the six scenarios; it can be observed, however, that extending the delay leads to better results (lower MSE) in general. The most effective training algorithm for large delays has turned out to be CGD. It gave the best results for the majority of scenarios under consideration, regardless of the number of neurons in the hidden layer (c.f. Table 2.) CGD resulted in the most efficient models for time series representing houses approved for construction (*permits-ind* and *permits-dev*, for delay of 150 in both cases), houses under construction (*initiated-ind* for delay of 100 and *initiated-dev* for delay of 150), as well as houses completed by developers (*completed-dev* for delay of 150). Only for the time series corresponding to houses completed by individuals (*completed-ind*) the best model was based on SCG training algorithm (for delay of 150).

This analysis demonstrates the critical impact of the number of neurons in the hidden layer and the delay parameter on the performance of various learning algorithms in NARX neural network modeling. It highlights the importance of fine-tuning these parameters for optimal predictive accuracy in different real estate development scenarios.



Figure 4: Testing Performance of NARX Models (left) and Results of the Jarque-Bera Test of the Normality of Residuals (right) for Various Learning Algorithms and Delays

Another aspect that was investigated to evaluate the model adequacy involved examining residuals, in particular, how their distributions conform to the normal distribution. As it was explained earlier, this conformity had been evaluated using the Jarque-Bera test. We denote a result of this test as " $H = 0$ " or " $H = 1$ ".  $H = 0$ indicates no grounds for rejecting the null hypothesis, i.e., the assumption of a normal distribution of residuals. In contrast,  $H = 1$  implies rejection of the null hypothesis, thereby challenging the assumption that the residuals follow a normal distribution.

Results of the Jarque-Bera test for the six datasets are presented in Fig. 4a – Fig. 4f on the right. Each plot shows *H* values for a given dataset and a given size of a NN hidden layer, *HL* (the one containing the best model cases), for different delays and training algorithms. Table 3 contains aggregate results (numbers of cases with the normality of residuals unconfirmed) for various training algorithms and *HL* sizes.

Results show that for all six datasets most of cases (configurations of delays, algorithms, and *HL* sizes) resulted in the normal distribution of residuals. The number of cases for which the normality of residuals was not confirmed was the highest for *initiated-ind* and *completed-ind* datasets (95 and 92 cases, respectively, at an average of 83). It indicates that time series representing houses being constructed and completed by individual investors are more unpredictable than the remaining time series. OSS and CGF algorithms were the least efficient in the analysed context (29 and 25 cases, respectively, at an average of 17.25).





Furthermore, a comparison of MSE values and Jarque-Bera test results for the analysed cases showed that some configurations of model parameters were able to provide high performance (low MSE values) but the hypothesis of the residuals normality could not be accepted.

# **SUMMARY**

This paper focused on analysing the dynamics of the housing construction market in Poland, employing the methodology of the NARX neural network. Three stages of the housing construction process have been taken into consideration (building permits, houses under construction, and completed houses) with a distinction made between individual investors and developers. The analysis focused on evaluating the NARX network model performance and tuning the model parameters. A careful selection and calibration of the model was conducted to adjust the model to the specificity of the analysed data. Special attention was paid to the size of the network's hidden layer and determination of the delay size of the model (the number of recent time intervals taken into consideration while doing predictions) for various training algorithms, aimed at the optimal modeling the nonlinear dependencies in the time series data.

Findings of the study provide significant insight into the efficiency of different NARX network configurations in modeling time series representing the numbers of houses accepted, initiated, and completed by individual and commercial investors. They emphasize the importance of selecting an appropriate neural network training algorithm, a hidden layer size, and a delay to obtain reliable forecasts. The most efficient algorithm, especially for high delays and regardless of the hidden layer size, was CGD (Conjugate Gradient with Powell/Beale Restarts).

The analysis revealed significant differences in trends and dynamics between the segments of developers and individual investors. The results indicate unique patterns in time series representing issued building permits, houses under constructions, and houses completed in both segments. These differences are crucial for understanding the specificity and mechanisms governing each of the markets.

## **REFERENCES**

- W. Breuer and B.I. Steininger. Recent trends in real estate research: a comparison of recent working papers and publications using machine learning algorithms. *J. of Business Economics*, *90*(7): 963–974, 2020.
- G. de Bondt, A. Gieseck, and M. Tujula. Household wealth and consumption in the euro area. *ECB Economic Bulletin*, 1, 2020.
- P. Frącz, I. Dąbrowski, D. Wotzka, D. Zmarzły, and Ł. Mach. Identification of differences in the seasonality of the developer and individual housing market as a basis for its sustainable development. *Buildings*, 13(2), 2023.
- L. Gabrielli, A.G. Ruggeri, and M. Scarpa. Using Artificial Neural Networks to uncover real estate market transparency: The market value. In *ICCSA'2021*, pages 183–192, 2021. Springer International Publishing.
- J.E. Gomez-Gonzalez, J. Hirs-Garzón, S. Sanin-Restrepo, and J.M. Uribe. Financial and macroeconomic uncertainties and real estate markets. *Eastern Economic Journal*, *50*(1): 29–53, 2024.
- A. Grybauskas, V. Pilinkienė, and A. Stundžienė. Predictive analytics using Big Data for the real estate market during the COVID-19 pandemic. *Journal of Big Data*, *8*(1): 105, 2021.
- GUS. Główny Urząd Statystyczny Publications, 2023. https://stat.gov.pl/publikacje/publikacje-a-z.
- L. Hong. The dynamic relationship between real estate investment and economic growth: Evidence from Prefecture City Panel Data in China. *IERI Procedia*, *7*: 2–7, 2014.
- I.I. Kakulu. Qualitative research strategies and data analysis methods in real estate research – an innovative approach using the BB Model. In *Estate Management Department Workshop*, 2014.
- J.H. Lang, M. Behn, B. Jarmulska, and M. Lo Duca. Real estate markets, financial stability and macroprudential policy. *Macroprudential Bulletin*, European Central Bank, 2022.
- MathWorks Documentation Design Time Series NARX Feedback Neural Networks, https://www.mathworks. com/help/deeplearning/ref/narnet.html (access date: 08.01.2024).
- S.W. Shao, X. Huang, L.X. Xiao, and H. Liu. Exploring the impact of real estate policy on real estate trading using the time series analysis. *The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, 42: 1281–1287, 2020.
- Statista. Real estate market in Poland statistics  $\&$  facts, https://www.statista.com/topics/7893/real-estate-marketin-poland/#topicOverview (access date: 25.03.2024).
- D. Wotzka, G. Suchacka, Ł. Mach, P. Frącz, J. Foltys, and I. Maniu. Time series prediction for the housing construction market with the use of NARNET. In *Proc. ECMS'2023*, pages 284–290, 2023. ECMS.
- W. Xiong. The impact of the real estate market on labor productivity. In *Journal of Education, Humanities and Social Sciences*, 14: 50–53, 2023.

Disclaimer: The paper presents personal opinions of the authors and does not necessarily reflect the official position of the Narodowy Bank Polski.

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