

# HYBRID FLOW SHOP SCHEDULING OF AUTOMOTIVE PARTS

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## KEYWORDS

Automotive parts, Hybrid flow shop scheduling,  
Optimal production schedule

## ABSTRACT

Flow shop scheduling problem is a type of scheduling dealing with sequencing jobs on a set of machines in compliance with predetermined processing orders. Each production stage to be scheduled in typical flow shop scheduling contains only one machine. However, in automotive part industry, many parts are produced in sequential flow shop containing more than one machines in each production stage. This circumstance cannot apply the existing method of flow shop scheduling. The objective of this research is to schedule the production process of automotive parts. The feature production is hybrid flow shop which consists of two-stages. In each stage, there are several manufacturing machines and each machine can produce more than one product. Thus, production scheduling is a complex problem. This paper, therefore, develops mathematical model to solve the hybrid flow shop production scheduling under different constraints of each machine. The setup time and production time of each machine can be different for each part. The solution for the experimental data sets from an automotive part manufacturer reveals that the process time can be reduced by 34.29%.

## INTRODUCTION

Automotive industry is a very important sector for the country's economy since it generates trade and financial inflows to the country. Automotive part (auto part) manufacturers play an important role in the industry to supply parts for vehicle manufacturers. The response time of auto part manufacturers, which is the total amount of time the manufacturers takes to respond to the orders of auto parts, greatly affects the vehicle production. Responding to vehicle manufacturer demands, then is the goal of automotive parts manufacturers. They have to plan their productions and schedule the machine operations to ensure the shortest total completion time for all orders. Typically, the production type of the automotive parts manufacturers is flow shop where the processes are in a predetermined processing order; one process must be completed before another. The machine

scheduling in the auto part manufacturer requires flow shop scheduling.

In flow shop scheduling the jobs must be produced through the first, second and the following stages. This scheduling problem is considered easy if there is only one machine for each production stage. The typical objective of flow shop scheduling is to minimize the makespan, i.e. to find the minimum total time needed to finish all of the production orders. Hence, the sequencing is decided for the scheduling problem. Typically, the flow shop scheduling deals with scheduling a number of jobs on different stages which contain only one machine on each stage. If each stage consists of many machines working in parallel, this problem is called hybrid flow shop scheduling (Choi et al. 2009). The scheduling problem becomes complicated which assigning and sequencing are required. The previous studies regarding the hybrid flow shop scheduling employ heuristics approaches to schedule the production. For example, Vignier et al. (1996) applies a branch and bound based algorithm to schedule jobs in multi-stages flow shop to minimize the makespan. Watanakich (2001) studies a two-stage hybrid flow shop scheduling with machine setup time, and solved the problem using a heuristic. He presents a two phase heuristic approach; constructing a schedule and assigning jobs with setup time consideration. This represents a difference between regular and hybrid flow shop scheduling. Wong et al. (2001) propose a genetic algorithm to schedule cutting and sewing operations in a manufacturer. Mallikarjuna et al. (2013) apply tabu search algorithm to complete flow shop scheduling. Puck-In (2014) tries to solve the scheduling problem by applying genetic algorithm and hybrid local search to minimize the makespan. It can be seen that most of previous studies apply heuristic algorithm to solve the hybrid flow shop scheduling in order that the makespan are minimized. However, the heuristic approaches do not typically guarantee the optimal solution for the problem.

This paper intends to present a mathematical formulation to solve a two-stage hybrid flow shop scheduling with the job and time constraints in order to achieve minimum makespan of all customer orders. Then the mathematical formulation is validated by applying the formulation to solve the hybrid scheduling in a case manufacturer.

The organization of this paper is as the following. The next section describes the hybrid flow shop scheduling problem. Then the generic mathematical formulation for hybrid flow shop scheduling is presented as a binary integer programming. After that, a numerical example of a case auto part manufacturer is presented to illustrate an application of the formulation to assign jobs to facilities and sequence the jobs. Finally, the conclusion and future research are presented.

## PROBLEM DESCRIPTION

This scheduling problem is a two-stage hybrid flow shop. In this flow processes, the jobs can be different types but they must be produced through the first and second stages. There are several non-identical parallel machines in each stage which some jobs cannot be produced at some machines. Also, the processing time of each job at each stage can be different when it is produced at different machines.

This problem studies the assigning and sequencing of jobs for each stage of two flow processes. The job must be accomplished and produced at a particular machines and specific sequence. The job  $i$  must be produced through the first stage using machine  $j$  in the sequence  $l$  and the second stage using machine  $k$  in the sequence  $l$  as shown in Figure 1.

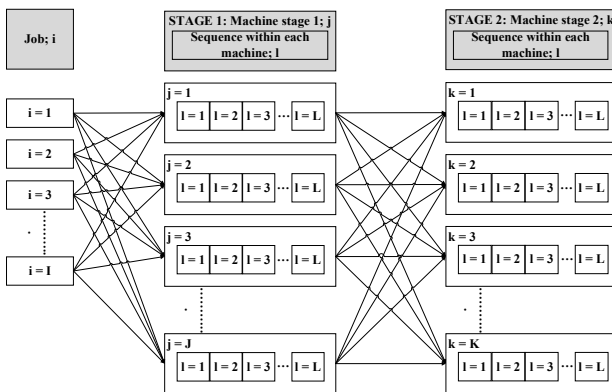


Figure 1: Hybrid Flow Shop Scheduling

## MATHEMATICAL FORMULATION

The section describes the mathematical model formulation for this hybrid flow shop with various constraints. The objective is to minimize the makespan in order to find the optimal scheduling consisting of assigning and sequencing.

### Indices

- $i$  job :  $i = 1, 2, 3, \dots, I$
- $j$  machine at  $j$  stage 1:  $j = 1, 2, 3, \dots, J$
- $k$  machine  $k$  at stage 2:  $k = 1, 2, 3, \dots, K$
- $l$  sequence of job:  $l = 1, 2, 3, \dots, L$

### Parameters

- $P_{ij}$  processing time of job  $i$  processed at machine  $j$  in stage 1
- $P_{ik}$  processing time of job  $i$  processed at machine  $k$  in stage 2
- $S_{ij}$  setup time of job  $i$  processed at machine  $j$  in stage 1
- $S_{ik}$  setup time of job  $i$  processed at machine  $k$  in stage 2
- $ST_{ijl}$  starting time of job  $i$  processed at machine  $j$  on sequence  $l$
- $ST_{ikl}$  starting time of job  $i$  processed at machine  $k$  on sequence  $l$
- $ET_{ijl}$  completion time of job  $i$  processed at machine  $j$  on sequence  $l$
- $ET_{ikl}$  completion time of job  $i$  processed at machine  $k$  on sequence  $l$
- $C_{ijl}$  total time of job  $i$  processed at machine  $j$  on sequence  $l$
- $C_{ikl}$  total time of job  $i$  processed at machine  $k$  on sequence  $l$

### Decision Variables

- $X_{ijl} = 1$  if job  $i$  is assigned at machine  $j$  on sequence  $l$ ; and 0 otherwise
- $Y_{ikl} = 1$  if job  $i$  is assigned at machine  $k$  on sequence  $l$ ; and 0 otherwise

### Dependent Variables

- $(C_{max})_{ik}$  total time of job  $i$  processed at the last sequence of machine  $k$

### Objective Function

The objective function for the scheduling is to minimize the makespan for all jobs.

$$\text{Minimize } Z = C_{max} \quad (1)$$

$$\text{where } C_{max} = \text{Max}\{E_{ikl}\} ; \forall i, \forall k$$

### Constraints

1. Constraints regarding the job assigned:

For each job, there should be only one job to be processed at machine  $j$  in sequence  $l$ .

$$\sum_{i=1}^I X_{ijl} = 1; \forall j, \forall l \quad (2)$$

Similarly, for each job, there should be only one job to be processed at machine  $k$  in sequence  $l$ .

$$\sum_{i=1}^I Y_{ikl} = 1; \forall k, \forall l \quad (3)$$

All the decision variables are binary.

$$X_{ijl}, Y_{ikl} \in \{0, 1\}; \forall i, \forall j, \forall k, \forall l \quad (4)$$

If there is any job that cannot be processed at a particular machine, that decision variable equals to 0. For example, if the jobs number 1 to 5 cannot be processed at machine 5 or stage 1, the decision variable  $X_{i5l}$  equals to 0:

$$X_{ijl} = 0; \forall l, i = 1, 2, 3, 4, 5 \quad (5)$$

## 2. Constraints related to time

The starting time of job  $i$  processed at machine  $j$  on the first sequence in stage 1 equals to 0.

$$ST_{ij1} = 0; \forall i, \forall j \quad (6)$$

The starting time of job  $i$  processed at machine  $j$  in sequence  $l$  equals to the completion time of its immediate predecessor job  $i$  processed at machine  $j$ .

$$ST_{ijl} = ET_{ij(l-1)}; \forall i, \forall j, l = 2, \dots, L \quad (7)$$

The starting time of job  $i$  processed at machine  $k$  in sequence  $l$  must greater than or equal to the completion time of job  $i$  processed at machine  $j$  in sequence  $l$ .

$$Y_{ikl}ST_{ikl} \geq E_{ijl}X_{ijl}; \forall i, \forall k, \forall l \quad (8)$$

The completion time of job  $i$  processed at machine  $j$  in sequence  $l$  equals to the summation of starting time of job  $i$ , set up time and processing time at machine  $j$ .

$$E_{ijl} = (ST_{ijl}X_{ijl}) + (S_{ij} + P_{ij})X_{ijl}; \forall i, \forall j, \forall l \quad (9)$$

The completion time of job  $i$  processed at machine  $k$  on sequence  $l$  equals to the summation of starting time of job  $i$ , set up time and processing time at machine  $k$ .

$$E_{ikl} = (ST_{ikl}X_{ikl}) + (S_{ik} + P_{ik})X_{ikl}; \forall i, \forall k, \forall l \quad (10)$$

## NUMERICAL EXAMPLE

The aforementioned mathematical formulation can be applied to the case of an automotive part manufacturer to solve the scheduling problems in the factory. The main processes in the case factory are metal cutting processes which machine automotive parts as per customer orders including washer and washer 5th gear thrust. The production of the two parts into the production flow shop is currently scheduled by assigning the job to the idle machines without scheduling plan. This results in long makespan and tardy jobs. The parts of washer and washer 5th gear thrust are often tardy. This needs to be change by planning the scheduling in advance.

The major machining processes for washers and washer 5th gear thrusts to be studied consists of 2 stages; Cutting and Turning processes. Cutting and Turning contain 5 and 3 machines, respectively. The problem can be depicted in Figure 2.

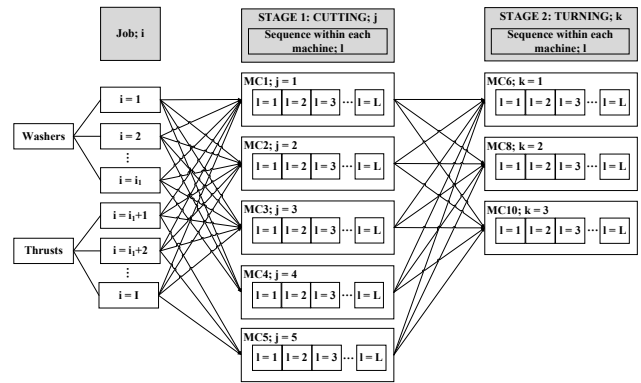


Figure 2: The case scheduling problem

The machines in each stage are interchangeable. There are only little exception regarding the selection of machines as presented in Table 1.

Table 1: Exceptions of machine selection

Stage	Machine no.	Product A Washer	Product B Thrust
Stage 1 Cutting	1	✓	✓
	2	✓	✓
	3	✓	✓
	4	✓	
	5		✓
Stage 2 Turning	6	✓	✓
	8	✓	✓
	10	✓	✓

In order to schedule these jobs, the job orders for the products must be grouped and assigned the job numbers. The example of the case contains 14 jobs; 7 jobs for washers and the rest for washer 5th gear thrust. Jobs of washers are assigned the job numbers 1 to 7, while jobs of thrusts are assigned numbers 8 to 14. The information regarding orders, number of pieces and processing time of each order on each machine are presented in Table 2. The setup time for each machine is 1 hour per a changeover.

Table 2: Jobs and their processing times

Product	Job (i)	Pieces	Processing time (hours)									
			Cutting					Turning				
			1	2	3	4	5	6	8	10		
Washer	1	450	7	3	6	6	0	5	3	1		
	2	552	3	8	7	4	0	6	3	2		
	3	487	7	2	6	3	0	5	3	5		
	4	650	10	10	9	2	0	7	4	3		
	5	500	8	8	7	2	0	3	5	5		
	6	480	7	7	6	2	0	3	5	5		
	7	378	6	6	5	1	0	4	2	4		
Washer 5th gear thrust	8	442	7	5	2	0	6	5	2	4		
	9	398	6	5	5	0	4	5	2	4		
	10	375	3	5	3	0	5	2	4	3		
	11	426	7	4	2	0	6	2	4	3		
	12	500	8	6	3	0	7	2	5	4		
	13	415	6	5	2	0	6	4	2	3		
	14	387	6	4	2	0	5	4	1	3		

Though the machines for each stage are interchangeable, the processing time on different machines are different. For example, Job 1 of 450 washers can be processed on machine no. 1, 2, 3, and 4. It takes 7 hours to complete 450 washers on machine no.1, while it takes only 3, 6, and 6 hours on machine 2, 3, and 4, respectively. Therefore, the selection of machines affects the makespan for all jobs. And it eventually affects the utilization of machines.

The previous scheduling technique used in this case factory yielded 35 hours makespan of scheduling for 14 jobs. The gantt chart to present the makespan of previous scheduling technique can be depicted in Figure 3.

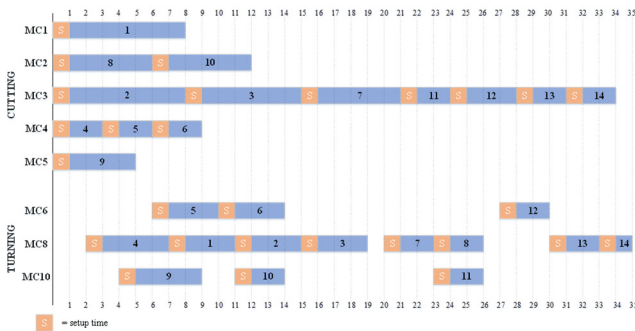


Figure 3: Scheduling applying previous scheduling technique

The aforementioned formulation can be used to shorten the makespan for all the 14 jobs.

### Results

The mathematical formulation of the case is applied to the case to make a decision for the scheduling of 7 orders of washers and 7 orders of washer 5th gear thrusts over one-week period (Table 2). The formulation is then solved by the Premium Excel Solver. Using the data of processing time in Table 2, the suitable machine for each job can be selected. The selection of machines yields the total makespan of 23 hours which is the minimum numbers of makespan for the 14 jobs. The result of machine selection can be presented in Table 3.

Table 3: Jobs and selections of machines

Decision variables Job (i)	Processing time (hours)							
	Cutting				Turning			
	1	2	3	4	5	6	8	10
$X_{1j}$	1	0	I	0	0	0	0	0
	2	I	0	0	0	0	0	0
	3	0	I	0	0	0	0	0
	4	0	0	0	I	0	0	0
	5	0	0	0	I	0	0	0
	6	0	0	0	I	0	0	0
	7	0	0	0	I	0	0	0
	8	0	0	I	0	0	0	0
	9	0	0	0	0	I	0	0
	10	0	0	I	0	0	0	0
	11	0	0	I	0	0	0	0
	12	0	0	I	0	0	0	0
	13	0	0	I	0	0	0	0
	14	0	0	I	0	0	0	0

Table 3: Jobs and selections of machines (cont.)

Decision variables Job (i)	Processing time (hours)							
	Cutting				Turning			
	1	2	3	4	5	6	8	10
$Y_{ik}$	1	0	0	0	0	0	0	I
	2	0	0	0	0	0	0	I
	3	0	0	0	0	0	I	0
	4	0	0	0	0	0	0	I
	5	0	0	0	0	0	I	0
	6	0	0	0	0	0	I	0
	7	0	0	0	0	0	0	I
	8	0	0	0	0	0	0	I
	9	0	0	0	0	0	0	I
	10	0	0	0	0	0	0	I
	11	0	0	0	0	0	0	I
	12	0	0	0	0	0	0	I
	13	0	0	0	0	0	0	I
	14	0	0	0	0	0	0	I

I = Selected machine for the particular job  
0 = Cannot be considered

From Table 3, it can be seen that the mathematical formulation can be used to select proper machines that yield the minimum makespan. Job 1 is to be cut on machine 2 and turned on machine 10; Job 2 is to be cut on machine 1 and turned on machine 10; and so on. Then, the jobs that need to be processed on the same machine need to be sequenced. The sequence of jobs in all cutting and turning machines are presented in Table 4.

Table 4: Sequence of jobs in each machine

Job (i)	MC No.	Sequence						
		1	2	3	4	5	6	7
$X_{1j}$ Cutting	1	2	I	0	0	0	0	0
	2	1	I	0	0	0	0	0
	3	2	0	I	0	0	0	0
	4	4	I	0	0	0	0	0
	5	4	0	I	0	0	0	0
	6	4	0	0	I	0	0	0
	7	4	0	0	0	I	0	0
	8	3	I	0	0	0	0	0
	9	5	I	0	0	0	1	0
	10	3	0	I	0	0	0	0
	11	3	0	0	I	0	0	0
	12	3	0	0	0	I	0	0
	13	3	0	0	0	0	I	0
	14	3	0	0	0	0	0	I
$Y_{ik}$ Turning	1	10	0	I	0	0	0	0
	2	10	0	0	I	0	0	0
	3	8	0	0	0	I	0	0
	4	10	0	0	0	I	0	0
	5	6	0	0	0	0	I	0
	6	6	0	0	0	0	0	I
	7	8	0	0	0	0	I	0
	8	8	0	I	0	0	0	0
	9	8	0	0	I	0	0	0
	10	6	0	0	I	0	0	0
	11	6	0	0	0	I	0	0
	12	6	0	0	0	0	0	I
	13	10	0	0	0	0	0	I
	14	8	0	0	0	0	0	0

Following the sequence of jobs processed at each machine presented in Table 4, the Gantt Chart to present the makespan for all 14 jobs can be depicted in Figure 4.

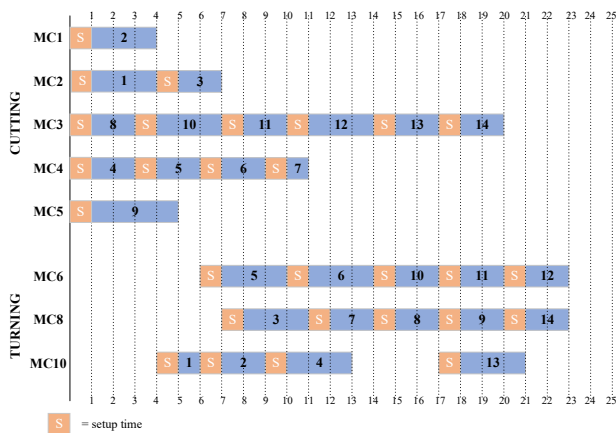


Figure 4: New scheduling applying created mathematical formulation

The result shows the optimal schedule of all the jobs in the two-stage hybrid flow shop by minimizing the makespan. The total makespan of all the 14 jobs is 23 hours. The scheduling in Figure 4 also informs starting, ending, and processing times for each job on each machine. The makespan of scheduling applying the mathematical formulation initiated in this paper is shorter than the 35 hours makespan of previous scheduling method (Figure 3) or 34.29% reduction in the makespan, regardless of longer total time in some jobs.

## CONCLUSION

This paper attempts to formulate the scheduling technique for hybrid flow shop production. It can be seen from the case that this mathematical model has a potential to create the optimal schedule for the two-stage flow shop that contains multiple machines in each process. This scheduling methodology concurrently considers the processes of 2 stages to ensure the minimum makespan of all the jobs. It is simply because the final makespan of all the jobs depends on the completion time of the last job

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on stage 2, whereas the starting time of stage 2 relies on the completion time of stage 1. The 2 stages must be considered simultaneously in order to obtain the optimal solution for the scheduling. This mathematical model can be considered useful since it can assign jobs to proper machines and sequence the jobs for each machine to create the optimal production schedule. For future research, this scheduling technique can be expanded to incorporate 3 or more stage production which is typical cases in many manufacturing industry, especially the automotive part industry.

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